Measuring the Welfare Gains of Technological Adoption: The Case of Steam Power in the Canadian Manufacturing Sector in the Late Nineteenth Century

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Abstract

This paper develops a methodology for measuring the aggregate welfare effect of technological adoption. Starting from a closed economy version of the Melitz (2003) model, I endogenize firms' decision to adopt a productivity enhancing technology. Firms are heterogeneous in productivity and the presence of a fixed cost of adoption results in only the most productive firms adopting the technology. The theoretical model is used to derive a sufficient statistic that measures the aggregate welfare gains from the introduction and adoption of a new technology. In the empirical application of the paper I use nineteenth century firm-level data from the Canadian manufacturing sector to estimate the welfare effects of the introduction of steam power. The results indicate that the aggregate welfare gain from steam power adoption ranged from 5.62-9.96%.

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1 Introduction

This paper develops and estimates the aggregate welfare effects of steam power adoption in the Canadian manufacturing sector in the late nineteenth century. I begin by showing that larger manufacturing firms were the first to use steam power during this transformational era of industrialization. This finding suggests that the rate of adoption and economic benefits associated with the use of steam power were dependent on size distribution of manufacturing establishments. Building on this intuition, I develop and estimate a model that explicitly accounts for the role of firm heterogeneity in the process of technological adoption.

In the spirit of Melitz (2003), I develop a model where firms are heterogeneous in productivity and face a fixed cost to adopt steam power. I solve the model and compare equilibrium in an economy without steam power to an economy where steam power is adopted by a subset of the largest and most productive firms. The introduction of steam power results in three effects on aggregate welfare. First, firms with the highest productivity adopt steam power which leads to an increase in aggregate productivity and a decrease in the aggregate price index. Second, the decrease in the price index results in the exit of the firms with the lowest productivity which further increases aggregate productivity. Therefore positive selection effects in both tails of the distribution result in aggregate welfare gains for the economy. However there is an offsetting third effect that reduces aggregate welfare resulting from the exit of the least productive firms. As in the Melitz (2003) model, each firm produces a distinct variety and consumers gain utility from having access to a larger variety of goods. Accordingly, the exit of the least productive firms following the introduction of steam power leads to a reduction in welfare resulting from the decrease in the set of goods available to consumers. I show that the aggregate welfare gains from steam power adoption are greater than the negative effects associated with a loss in product variety. I accomplish this by deriving a sufficient statistic that measures the aggregate welfare effects from steam power adoption and prove that this statistic is always positive.

In the empirical section of the paper I develop an empirical methodology for estimating the aggregate welfare gains from the introduction of steam power. For my empirical application I use firm-level data on value added and steam power adoption from the Canadian Censuses of Manufacturing in 1871 respectively. I find that the aggregate welfare gains from the introduction and adoption of steam power ranged from 5.62-9.96%.

There is a large literature on the adoption of steam and water power in the US manufacturing sector in the second half of the nineteenth century. The emphasis of the literature on this particular era stems from the rapid diffusion of steam power during this period, as documented by Hunter (1985). Atack et al. (1980) also analyzes the steam and water power diffusion in the nineteenth century by focusing on the spatial variation in the cost and rate of adoption in different regions of the US. Atack et al. (2008) focus on scale differences in mechanized power adoption and find that small US firms were less likely to adopt steam power than large firms. The authors also find that firms using steam or water power had higher labour productivity, although their results indicate that this productivity advantage was largely attributable to the higher levels of capital intensity in establishments with mechanized power. Using manuscript data for Ontario from the 1871 Canadian Census of Manufacturing, Bloomfield and Bloomfield (1989) find that establishments adopting water and steam power tended to be larger on average, with higher value added, more employees, and higher capital intensity. While this result suggests a parallel between the Canadian and US experience, Inwood and Keay (2012) find that the Canadian Manufacturing sector was slower to adopt steam power than the US. The authors also find that water and steam power provided a productivity advantage, as firms operating with steam or water power had 11% higher total factor productivity. My research contributes to this literature in several respects. To my knowledge, this paper is the first to develop a methodology for estimating the aggregate welfare effects of steam power adoption that accounts for the role of firm heterogeneity. While the empirical application I consider focuses on steam power in the late nineteenth century, the framework is equally applicable to the adoption of other technologies in other eras. Finally by using data from the Canadian manufacturing sector, my research contributes to the economic history literature on steam power in the nineteenth century. While there is a substantial literature on steam power adoption in the US and Europe in the nineteenth century, the understanding of steam power adoption in the Canadian economy during this era is less well understood.

The remainder of the paper is organized as follows: Section 2 describes the data and establishes some basic empirical results that guide the development of the theoretical and empirical framework. Section 3 presents the theoretical model, Section 4 presents the empirical application, and Section 5 concludes.

2 Steam Power and Productivity in Canadian Manufacturing in the Late Nineteenth Century

In this section I present several empirical results on the adoption of steam power in the Canadian manufacturing sector in 1871. This exercise serves two main purposes. First, the empirical findings in this section will be used to guide the development of the theoretical and empirical framework that is used in the remainder of the paper. Second, the results provide some basic stylized facts about steam power adoption in Canadian manufacturing that can be compared with the US experience during this era.

Three primary sources of data are used in this section. First is the Inwood and Keay (2012) firm-level sample of 1871 manufacturing establishments from the 1871 Canadian Census of Manufacturing. In total, the Inwood and Keay (2012) sample includes observations on 27,111 establishments. Inwood (1995) notes that after the reconstitution of multi-process establishments, the total number of establishments listed in the 1871 manuscripts is 40,761. Thus the 27,111 establishments in the sample represent approximately two-thirds of the total number of manufacturing establishments in Inwood's reconstituted version of the 1871 manuscripts. The Inwood and Keay (2012) sample includes all establishments from the 20 largest Canadian manufacturing industries, with the size of the industry being measured by the number of establishments in 1871. The sample does not include observations on firms from the most highly consolidated manufacturing industries. This feature limits the suitability of the Inwood and Keay (2012) sample for my analysis as a number of these consolidated industries were among the earliest to adopt water or steam power. For example, the distillery industry is not included in the Inwood and Keay (2008) sample since there were fewer than 20 distilleries in Canada in 1871. However all distilleries listed in the Census of Manufacturing were using either water or steam power in 1871.[1]

The second data source I use is the CANIND71 (2008) database, which is a digitized record of the universe of manufacturing establishments that were enumerated in the 1871 Manufacturing Census of Canada. The history of the CANIND71 database is documented in Bloomfield and Bloomfield (1989). While the CANIND71 database is comprehensive, there are measurement issues in the raw manuscript data that must be reconciled prior to analysis. In particular, Inwood (1995)

[1]Information on the Canadian distillery industry is sourced from the CANIND71 (2008) database.

notes that multi-process establishments were occasionally decomposed by enumerators into multiple manuscript entries based on the distinct industrial activities carried out by the firm. Inwood (1995) finds that approximately 10% of the census manuscript entries were establishments that had been decomposed by enumerators. Using the data available in the CANIND71 database, I follow Inwood's (1995) procedure for identifying and reconstituting multi-process establishments.[2] Reconstitution of the CANIND71 database reduces the number of establishments from 45,070 to 42,788. Whereas the limitations of the CANIN71 database prevent me from following precisely the same reconstitution strategy as Inwood (1995), the empirical analysis in this section is completed using both the CANIND71 and the Inwood and Keay (2012) sample to demonstrate the robustness of my findings for the Canadian manufacturing sector.

The third source of data used is the Atack and Bateman (1999) national representative sample from the 1870 US Census of Manufacturers. This sample includes 5,296 firm-level observations based on random sampling from each US state with surviving census enumeration records. I have included the Atack and Bateman (1999) sample in my analysis to benchmark the degree of steam power adoption in the Canadian economy in 1871 versus the US.

Table 1: Steam Power Adoption by Firm Size

Sample	Inwood, Keay (2012)	CANIND71	Atack, Batemen (1999)
<5 Employees	3.35%	2.61%	9.19%
	(0.180)	(0.159)	(0.289)
$5-10$ Employees	21.80%	22.10%	32.70%
	(0.413)	(0.415)	(0.470)
>10 Employees	25.60%	37.00%	19.60%
	(0.437)	(0.483)	(0.397)
Overall	5.79%	6.64%	16.90%
	(0.234)	(0.249)	(0.375)
Observations	27,111	42,788	5,296

Sample standard deviation estimates are in parentheses. Columns 2 and 3 are calculated using 1871 Canadian Census of Manufacturing firm-level data from the Inwood and Keay (2008) sample and the CANIND71 database respectively. Column 4 is calculated using 1870 US Census of Manufacturers firm-level data from the Atack and Bateman (1999) national representative sample.

[2] Inwood (1995, p. 364) uses the following reconstitution strategy: "The criteria for combining two or more entries into a larger and more complex firm are that they share a proprietor name within an enumeration division, that they appear immediately adjacent to each other in the manuscript schedule, and that an examination of the personal schedule 1 for the immediate area does not reveal the presence of two potential proprietors with the same name." I follow the same strategy as Inwood (1995), except that I omit the final criterion as I do not have access to the micro-data from schedule 1 of the 1871 Census of Canada.

Table 1 illustrates the positive relationship between steam power adoption and manufacturing establishment size in Canada and the US. The first three rows of Table 1 shows the percentage of firms using steam power conditional on being small (less than 5 employees), medium (5-10 employees), or large (more than 10 employees). In each dataset the percentage of establishments using steam power increases when moving from small to medium, and medium to large size firms. Table 1 also illustrates some important differences across the three datasets. The use of steam power among large establishments in the CANIND71 data is 11.5 percentage points higher than in the Inwood and Keay (2008) sample. This suggests that the large consolidated industries that are excluded from the Inwood and Keay (2008) had higher rates of steam adoption, as was the case for the distillery industry as discussed above. The final column in Table 1 presents the results from the Atack and Bateman (1999) sample and enables a comparison of steam power adoption in the US and Canadian manufacturing sectors. The results indicate that the overall rate of adoption was more than 10 percentage points higher in the US,[3] a finding that is consistent with the research of Inwood and Keay (2012) and Bloomfield and Bloomfield (1989). However, although the overall rate of adoption was higher in the US, the percentage of large manufacturing firms adopting steam power was lower in US than in Canada. Thus, the higher overall rate of steam power adoption in the US stems primarily from the higher rate of adoption among small and medium sized manufacturing firms in the US as compared with Canada.

The positive correlation between firm scale and steam power adoption can be rationalized by economies of scale, especially if one assumes that adopting steam power entails large fixed costs. If firms are heterogeneous in productivity it follows that only the largest and most productive firms will be able to finance steam power adoption. The regression results in Table 2 test the correlation between labour productivity and steam power adoption in the Canadian and US data.

[3] Inwood and Keay (2008) note that comparative analysis of the Canadian and US manufacturing sectors using nineteenth century census manuscript data requires careful consideration of the differences in the enumeration practices in each country. While the enumeration practices were similar in Canada and the US, one important difference was with respect to the treatment of very small manufacturing establishments. US enumerators were instructed not to include any firms with gross sales less than $500, whereas the Canadian enumerators were required to survey all manufacturing establishments regardless of size. To account for this difference in enumeration practice, I have recalculated the values in Table 1 after removing all establishments from the Inwood and Keay (2012) and CANIND71 database having less than $500 US dollars in gross sales. US dollars are converted to Canadian at the official exchange rate (1 US dollar could be exchanged for approximately 0.83 Canadian dollars in 1871 (Inwood and Keay, 2008)). Even after dropping these observations from the Canadian data the rate of steam power adoption is lower in Canada relative to the US. Specifically, the overall percentage point difference in steam power adoption favours the US by 9.2 and 7.3 percentage points when compared with the Inwood and Keay (2012) sample and CANIND71 database respectively. These results are available upon request.

The dependent variable is the natural logarithm of labour productivity (value added divided by employment weighted by months in operation). Value added is measured as gross sales minus the value of raw material inputs. Employment is measured as the sum of male, female and child labour workers weighted by months in operation.[4] I control for measurement error in the CANIND71 and Atack et al. (2008) data by removing the bottom and top 1% of the empirical distribution of labour productivity.[5] The capital-labour ratio is the natural logarithm of fixed capital divided by employment weighted by months in operation. Indicator variables for firms' use of steam and water power are included in all regressions.[6] All regressions also include indicator variables for industry type, district/county, and province/state. For the Inwood and Keay (2008) and Atack and Bateman (1999) sample I also include an indicator variable for urban centres.[7] The CANIND71 database does not report the population of census districts or sub districts so no urban indicator variable is included in the regression for that sample.

Table 2: Regression Analysis of Labour Productivity

Sample	Inwood, Keay (2008)	CANIND71	Attack, Batemen (1999)
Steam Power	0.0449**	0.0745***	0.0644
	(0.0191)	(0.0181)	(0.0654)
Water Power	-0.00446	0.0321*	-0.127
	(0.0205)	(0.0192)	(0.0814)
$5-10$ Employees	0.0105	0.0413***	0.0368
	(0.0157)	(0.0129)	(0.0625)
>10 Employees	0.00590	0.0436***	-0.0703
	(0.0170)	(0.0152)	(0.0649)
Capital-Labour Ratio	0.276***	0.211***	0.254***
	(0.00388)	(0.00358)	(0.0211)
Observations	26,997	37,670	2,252
R-Squared	0.366	0.330	0.597

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Firms with fewer than 5 employees are the reference group for the employment indicator variables. Indicator variables for industry type, district/county, and province/state are included in all specifications. An indicator variable for urban status is also included in the regression using the Inwood and Keay (2008) sample and the Atack and Bateman (1999) sample. A complete set of results for the indicator variables are available upon request.

[4]The Atack and Bateman (1999) sample size is greatly reduced due to the large number of missing observations for female and child employees.

[5]This filter is not applied to the Inwood and Keay (2008) data as it has already been cleaned by the authors.

[6]The CANIND database also records observations for a small number of firms that used wind power (53 observations), and for firms that used both water and steam power (157 observations). Indicator variables are also included in the CANIND71 regression analysis for these groups.

[7]Atack and Bateman (1999) identify urban counties as those having a population greater than 2,500. For the Inwood and Keay (2008) sample, I identify urban sub-districts as those having a population density greater than 1000 persons per square mile.

The regression results in Table 2 indicate that steam power was correlated with higher labour productivity in the Canadian manufacturing sector in 1871, holding constant the other controls variables in the regression analysis. The coefficient on the steam power variable is positive, and statistically significant at the 5% and 1% levels for the Inwood and Keay (2008) and CANIND71 samples respectively. This differs from the regression results for the Atack and Bateman (1999) US data where no statistically significant relationship is found between labour productivity and steam power. This finding is consistent with Atack et al. (2008) who find that the labour productivity advantage associated with steam and water power in the US was largely attributable to higher capital intensity. The results from the CANIND71 data also indicate weak evidence of a positive correlation between water power and labour productivity, however no statistically significant relationship is found in the other two samples. Atack et al. (2008) note that relative to water power, steam was a cheap and accessible power source and had the clear advantage of not being tied to a geographic location. This logic seems a fitting explanation to the finding that steam was more highly correlated with productivity then water in the Canadian manufacturing sector. Returns to scale are also apparent in the CANIND71 data as coefficients on the indicator variables for firms having $5-10$ and greater than 10 employees are both statistically significant at the 1% level. The coefficients show the strength of the correlation between firm size and labour productivity relative to the reference group, which is firms with fewer than 5 employees. The indicator variables for firm size are not statistically in regressions using the Atack and Bateman (1999) and Inwood and Keay (2008) samples.

The empirical analysis in this section can be summarized by two main results. First steam power adoption was associated with higher labour productivity in the Canadian manufacturing sector. In contrast to the US, this positive correlation persists even when capital intensity and several other control variables are included in the regression analysis. Second, in both Canada and the US steam power adoption in the late nineteenth century was more common among large manufacturing establishments. In the next section I develop a theoretical model of technological adoption that is consistent with these findings. I will then use this theoretical framework to estimate the welfare gains from the introduction and adoption of steam power in the Canadian economy during this era.

3 Theoretical Model

In this section I develop a methodology for measuring the aggregate welfare effects of steam power adoption. I consider a closed economy version of the Melitz (2003) model. Consumption goods are produced by monopolistically competitive firms that use labour as the sole input in production. Firms are heterogeneous in productivity and face fixed costs of entry and production. My model differs from Melitz (2003) in that I assume firms must pay a fixed cost to adopt steam power, which is modeled as a fixed factor in production.

There is a unit measure of identical households with CES utility:

$$U = \left(\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 1 \tag{1}$$

where $q(\omega)$ is the quantity of variety ω demanded by the representative household, σ is the elasticity of substitution between varieties, and Ω is the set of varieties produced. Each household has one unit of labor that is supplied inelastically to firms. Households also hold an equal share in the profits of intermediate and final goods producers.

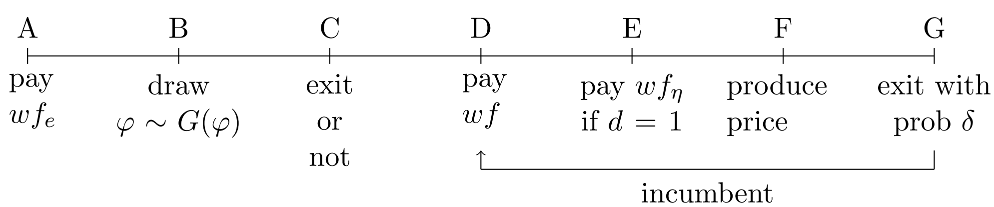

Figure 1: Firms' Timeline

Timing in the goods sector follows the framework used in Melitz (2003) and is summarized in Figure 1. At stage A, firms choose to enter the industry by paying a fixed cost f_e. All fixed costs are denoted in units of labour. After paying the entry cost, firms draw their productivity type, φ, from a Pareto distribution at stage B:[8]

$$G(\varphi) = 1 - \left(\frac{b}{\varphi}\right)^{\theta} \quad if\ \varphi \geq b,\ and\ 0\ otherwise. \quad \theta > \sigma - 1$$

where b is the scale parameter and θ is the scale parameter of the Pareto distribution. I assume

[8] I follow Arkolakis et al. (2012) in assuming that firm productivity follows a Pareto distribution.

that $\theta > \sigma - 1$ which ensures that the ex-post average productivity type of producers is finite. Upon realizing their type, firms face a decision at stage C to exit or remain in the industry. In each period, active firms must pay a fixed cost of production, f. Firms can choose to adopt steam power by paying a one time fixed cost F_η. Firms are indifferent between paying this one time fixed cost and paying an amortized portion of this payment equal to $f_\eta = \delta F_\eta$ in each period. This is because all uncertainty in the model is resolved after the firm draws its type, φ, and because there is no time discounting apart from that associated with the probability of exogenous exit. For notational convenience I model the fixed cost of steam power adoption using the per period payment representation. That is, at stage E firms can choose to adopt steam power by paying the per period fixed cost, f_η. In this paper I adopt the following convention to model the steam power adoption decision:

$$d = \begin{cases} 1 & \text{if firm adopts steam power.} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Firms subsequently make a pricing and production decision at stage F. Finally, at the end of each period firms are forced to exit with probability δ. The incumbent firm cycles over stages D through G until it realizes the exogenous exit shock.

Firms use the following technology in production:

$$q(\varphi) = \varphi(1 + d\eta)l, \quad \eta > 0 \tag{3}$$

The adoption of steam power is modeled as a fixed factor of production, and the magnitude of the productivity increase that results from using steam power is dictated by the parameter η.

I begin by considering the representative household's utility maximization problem:

$$\max_{\{q(\omega)\}\omega\in\Omega} \left(\int_{\omega\in\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

$$\textit{subject to}: \; w \geq \int_{\omega\in\Omega} q(\omega)p(\omega)d\omega,$$

where w is the wage and $p(\omega)$ is the price of variety ω.[9] Solving this problem for the representative

[9] I have imposed the equilibrium result of zero profits (in aggregate) to simplify the presentation of the household budget constraint.

household yields the demand for variety ω:

$$q(\omega) = \frac{wP^{\sigma-1}}{p(\omega)^{\sigma}}, \tag{4}$$

where P is the price index:

$$P \equiv \left(\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

Next I consider the problem of firms by working backward from stage G. Monopolistic competition implies that a firm of type φ will post the price:

$$p(\varphi; d) = \frac{\sigma}{\sigma-1} \frac{w}{(1+d\eta)\varphi} \tag{5}$$

In equation (5) I adopt the convention of writing $p(\varphi; d)$ to emphasize the dependence of the pricing rule on the decision to adopt steam power. Combining the firm's pricing rule with the household demand yields the following revenue formula:

$$r(\varphi; d) = w \left(\frac{P}{p(\varphi; d)} \right)^{\sigma-1} \tag{6}$$

The profit of a firm of type φ can be expressed as a function of the firm's revenue as follows:

$$\pi(\varphi; d) = \frac{r(\varphi; d)}{\sigma} - dwf_{\eta} - wf \tag{7}$$

The presence of the fixed cost, f_{η}, implies that only the most productive firms adopt steam power at stage E. Define φ^{1*} as the cut-off, such that a firm with this productivity level is just indifferent with respect to the adoption of steam power. That is, $\pi(\varphi_{1*}; 1) = \pi(\varphi^{1*}; 0)$, which from the profit equation (7) implies:

$$r(\varphi^{1*}; 0) = \frac{wf_{\eta}\sigma}{(1+\eta)^{\sigma-1} - 1} \quad \Longleftrightarrow \quad r(\varphi^{1*}; 1) = \frac{(1+\eta)^{\sigma-1} wf_{\eta}\sigma}{(1+\eta)^{\sigma-1} - 1} \tag{8}$$

Define $\tilde{\varphi}^1$ as the weighted harmonic mean value of φ for the subset of producers that adopt steam power:

$$\tilde{\varphi}^1 = \left(\int_{\varphi^{1*}}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi^{1*})} d\varphi \right)^{\frac{1}{\sigma-1}}$$

Following the same approach as in Melitz (2003), the revenue of a firm of type $\tilde{\varphi}^1$ can be related to the revenue of a firm at the cut-off, φ^{1*}, which is defined by the rightmost equation in (8). Using this approach and subbing $r(\tilde{\varphi}^1; 1)$ into the profit equation implies that the profit of a firm of type $\tilde{\varphi}^1$ is:

$$\frac{\pi(\tilde{\varphi}^1; 1)}{w} = \left(\frac{\tilde{\varphi}^1}{\varphi^{1*}}\right)^{\sigma-1} \frac{(1+\eta)^{\sigma-1}}{(1+\eta)^{\sigma-1} - 1} f_\eta - f_\eta - f, \qquad (ZCP^1),$$

where $\pi(\tilde{\varphi}^1; 1)/w$ is the average level of profit conditional on a firm adopting steam power, denoted in units of labour. Equation (ZCP^1) is the *zero-cut-off-profit* condition for steam power adoption.

I now turn to the entry decision at stage C. Define φ^{0*} as the cut-off, such that a firm with this productivity level is just indifferent between entering and exiting after learning its productivity type. I assume that $\varphi^{0*} < \varphi^{1*}$, so that a subset of firms, $\varphi \in \left[\varphi^{0*},\ \varphi^{1*}\right)$, produce without steam power. By definition a firm that draws φ^{0*} makes zero profit, $\pi(\varphi^{0*}; 0) = 0$. From the profit equation (7) it follows that:

$$r(\varphi^{0*}; 0) = wf\sigma \tag{9}$$

Define $\tilde{\varphi}^0$ as the weighted harmonic mean value of φ for the subset of active producers without steam power:

$$\tilde{\varphi}^0 = \left(\int_{\varphi^{0*}}^{\varphi^{1*}} \varphi^{\sigma-1} \frac{g(\varphi)}{G(\varphi^{1*}) - G(\varphi^{0*})} d\varphi\right)^{\frac{1}{\sigma-1}}$$

The revenue of a firm of type $\tilde{\varphi}^0$ can be derived relative to a firm of type φ^{0*}, which is defined by equation (9). Following this approach and subbing $r(\tilde{\varphi}^0; 0)$ into the profit equation implies that the profit of a firm of type $\tilde{\varphi}^0$ is:

$$\frac{\pi(\tilde{\varphi}^0; 0)}{w} = f\left(\frac{\tilde{\varphi}^0}{\varphi^{0*}}\right)^{\sigma-1} - f, \qquad (ZCP^0)$$

where $\pi(\tilde{\varphi}^0; 0)/w$ is the average profit of an active firm that does not adopt steam power, denoted in units of labour. Equation (ZCP^0) is the *zero-cut-off-profit* condition for entry.

In order to solve the model it is necessary to derive an equation that relates the entry cut-off, φ^{0*}, to the cut-off for steam power adoption, φ^{1*}. By definition of the revenue function:

$$\frac{r(\varphi^{1*}; 0)}{r(\varphi^{0*}; 0)} = \left(\frac{\varphi^{1*}}{\varphi^{0*}}\right)^{\sigma-1} \iff r(\varphi^{1*}; 0) = \left(\frac{\varphi^{1*}}{\varphi^{0*}}\right)^{\sigma-1} wf\sigma, \tag{10}$$

where the rightmost equation follows from equation (9). Combining equation (10) with the leftmost equation in (8) yields the *relative-cut-off* (RC) condition:

$$\varphi^{1*} = \varphi^{0*} \left(\frac{f_\eta}{f((1+\eta)^\sigma - 1)} \right)^{\frac{1}{\sigma-1}} \qquad (RC)$$

My earlier noted assumption, that $\varphi^{1*} > \varphi^{0*}$, requires that $f_\eta > f((1+\eta)^\sigma - 1)$.

Define $\tilde{\pi}/w$ as the expected profit conditional upon entry. Imposing the assumption that φ is drawn from the Pareto distribution as in (2), and combining the conditions (RC), (ZCP^0), and (ZCP^1) yields:

$$\frac{\tilde{\pi}}{w} = \frac{E(\pi|\phi \geq \phi^{0*})}{w} = \frac{f(\sigma-1)}{\theta-\sigma+1} \left(1 + \left(\frac{f}{f_\eta} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \left((1+\eta)^{\sigma-1} - 1\right)^{\frac{\theta}{\sigma-1}} \right) \qquad (ZCP)$$

Finally at stage A firms will enter the industry up until the point that expected profits are equal to the fixed cost of entry, $(1 - G(\varphi^*))\,\tilde{\pi}/\delta = w f_e$. This free entry condition can be rewritten:

$$\frac{\tilde{\pi}}{w} = \frac{\delta f_e}{1 - G(\varphi^*)} = \delta f_e \left(\frac{\varphi^{0*}}{b} \right)^\theta \qquad (FE)$$

The equilibrium entry cut-off, φ^{0*}, can be solved by combining equations (ZCP) and (FE):

$$\varphi^{0*} = b \left(\frac{f(\sigma-1)}{\delta f_e(\theta-\sigma+1)} \right)^{\frac{1}{\theta}} \left(1 + \left(\frac{f}{f_\eta} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \left((1+\eta)^{\sigma-1} - 1\right)^{\frac{\theta}{\sigma-1}} \right)^{\frac{1}{\theta}} \qquad (11)$$

The equilibrium entry cut-off is decreasing in the fixed cost of steam power adoption, f_η, and increasing η, which is the parameter that governs the productivity increase from adopting steam power. Define φ_a^* as the equilibrium cut-off when steam power is not available in the economy. Solving the model without steam power yields:

$$\varphi_a^* = b \left(\frac{f(\sigma-1)}{\delta f_e(\theta-\sigma+1)} \right)^{\frac{1}{\theta}} = \lim_{f_\eta \to \infty} \varphi^{0*} = \lim_{\eta \to 1} \varphi^{0*}$$

Intuitively, when the cost of steam power adoption becomes prohibitively high, or when the productivity increase associated with steam power adoption is small, the equilibrium entry cut-off approaches its value in the economy without steam power.

The introduction of steam power results in three effects on aggregate welfare, which are depicted graphically in Figure 2. First, firms with productivity higher than φ^{1*} adopt steam power, which leads to an increase in aggregate productivity and a decrease in the aggregate price index. Second, the decrease in the price index results in the exit of the firms with productivity below φ^{0*}, which further increases aggregate productivity. These two positive selection effects increase aggregate welfare. Third, the exit of the least productive firms leads to a reduction in the set of products available to consumers, which decreases aggregate welfare as consumers gain utility from product variety.

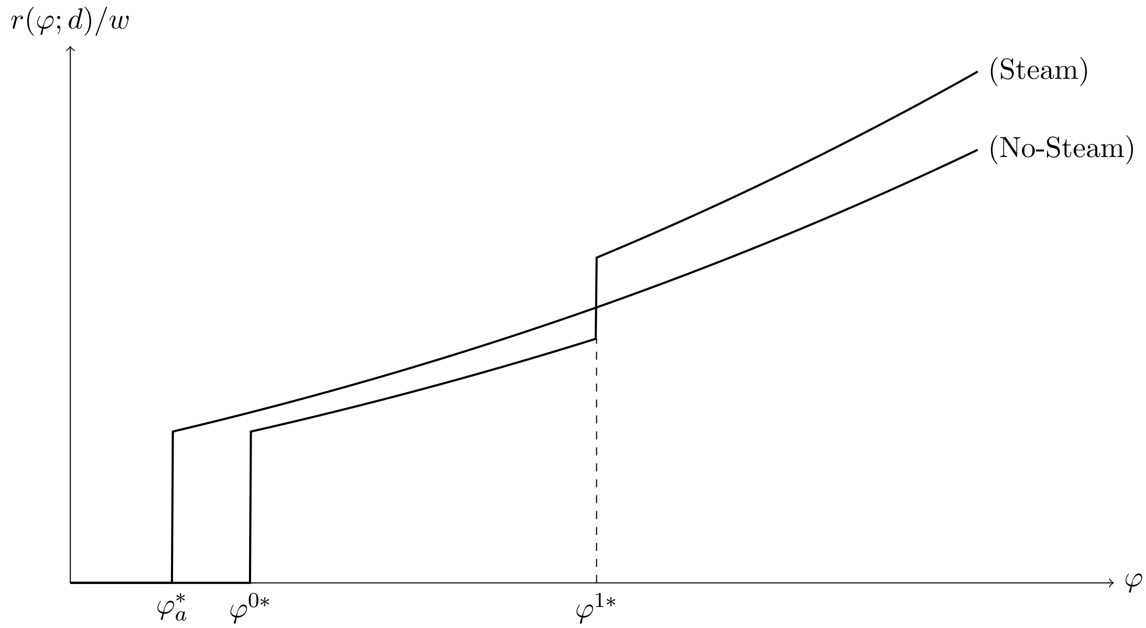

Figure 2: Reallocation of Market Share in Response to the Introduction of Steam Power

I now derive a sufficient statistic that summarizes the aggregate welfare effects of introduction and adoption of steam power in the model economy presented above.[10] Through the derivation of this statistic I prove that the aggregate welfare effects from steam power adoption are always positive. This sufficient statistic can also be used to empirically estimate the aggregate welfare effect from the introduction of steam power, which is completed in Section 4 for the case of the Canadian manufacturing sector in the late nineteenth century.

Under CES utility the appropriate measure of welfare is the real wage, $\mathbb{W} = w/P$, where $\mathbb{W}$ denotes welfare, w and P are the nominal wage and aggregate price index respectively. Welfare in

[10] A number of the steps in the derivation below follow Feenstra (2010).

the economy without steam power is defined analogously, $\mathbb{W}_a = w_a/P_a$.

Proposition 1: The increase in welfare from the introduction and adoption of steam power can be measured by the increase in the equilibrium entry cut-off. That is:

$$\frac{\mathbb{W}}{\mathbb{W}_a} = \frac{w}{w_a}\frac{P_a}{P} = \frac{\varphi^{0*}}{\varphi_a^*} = \left(1 + \left(\frac{f}{f_\eta}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} \left((1+\eta)^{\sigma-1} - 1\right)^{\frac{\theta}{\sigma-1}}\right)^{\frac{1}{\theta}} \tag{12}$$

Proof: It is convenient to invert the demand curve and write the revenue of a firm without steam power as a function of demand:

$$r(\varphi;0) = P\left(\frac{w}{P}\right)^{\frac{1}{\sigma}} q(\varphi;0)^{\frac{\sigma-1}{\sigma}} \tag{13}$$

The above expression also defines firm-level revenue in the economy without steam power. Evaluating equation (13) at φ^{0*} and φ_a^*, and using equation (9) to evaluate ratio $r(\varphi^{0*};0)/r(\varphi_a^*)$ yields:[11]

$$\frac{w}{w_a}\frac{P_a}{P} = \frac{q(\varphi^{0*};0)}{q(\varphi_a^*)} \tag{14}$$

Finally, I use the fact that equation (8) and the definition of the equilibrium price in equation (5) imply $q(\varphi^{0*};0) = (\sigma-1)f\varphi^{0*}$. In the economy without steam power the same line or reasoning implies $q(\varphi_a^*) = (\sigma-1)f\varphi_a^*$. Substituting these expressions into equation (14) completes the proof:

$$\frac{w}{w_a}\frac{P_a}{P} = \frac{\varphi^{0*}}{\varphi_a^*} = \left(1 + \left(\frac{f}{f_\eta}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} \left((1+\eta)^{\sigma-1} - 1\right)^{\frac{\theta}{\sigma-1}}\right)^{\frac{1}{\theta}},$$

where the last equality follows from the equilibrium values of the entry cut-offs in equations (11) and (12). □

Estimating the welfare effects of steam power adoption requires an estimate of the scale parameter of the Pareto distribution, θ, and an estimate of the term in large parenthesis in the rightmost expression in equation (12). I now illustrate how a convenient expression for the latter term can be derived from the labour market clearing condition:

$$1 = M_e f_e + \int_{\varphi^{0*}}^{\varphi^{1*}} \left(\frac{q(\varphi;0)}{\varphi} + f\right) M \frac{g(\varphi)}{1-G(\varphi^{0*})} d\varphi + \int_{\varphi^{1*}}^{\infty} \left(\frac{q(\varphi;1)}{(1+\eta)\varphi} + f + f_\eta\right) M \frac{g(\varphi)}{1-G(\varphi^{0*})} d\varphi, \quad (LMC)$$

[11]Equation in (9) continues to hold in the economy without steam power. That is, $r(\varphi_a^*) = w_a f\sigma$.

where, M_e is the measure of entrant firms. Next I multiply equation (LMC) by w and substitute in the following expression $wq(\varphi;d)/\varphi = (1+d\eta)r(\varphi;d)(\sigma-1)/\sigma$, which is derived from the definition of revenue and the price equation (5). Following these substitutions equation (LMC) can be re-written:

$$w = M_e f_e + Mf + \frac{1-G(\varphi^{1*})}{1-G(\varphi^{0*})}Mf_\eta + \frac{\sigma-1}{\sigma}\int_{\varphi^{0*}}^{\infty} r(\varphi,d)M\frac{g(\varphi)}{1-G(\varphi^{0*})}d\varphi,$$

$$= M_e f_e + Mf + \frac{1-G(\varphi^{1*})}{1-G(\varphi^{0*})}Mf_\eta + \frac{\sigma-1}{\sigma}w, \tag{15}$$

where the final equation follows from the definition of GDP.

To solve for M_e, I adopt the stationary assumption that the measure of firms entering in any period is equal to the measure that exit that, $(1-G(\varphi^{0*}))M_e = \delta M$. Using the stationary assumption to solve for M_e and subbing this value along with the equilibrium values of the cut-offs, φ^{0*} and φ^{1*}, into (15) yields:

$$1 + \left(\frac{f}{f_\eta}\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\left((1+\eta)^{\sigma-1}-1\right)^{\frac{\theta}{\sigma-1}} = \frac{(\theta-\sigma+1)}{\sigma f M\theta} = \frac{w(\theta-\sigma+1)}{r(\varphi^{0*},0)M\theta} = \frac{R}{r(\tilde{\varphi},0)M}, \tag{16}$$

where R denotes aggregate sales and $r(\tilde{\varphi},0)$ is defined as follows:

$$r(\tilde{\varphi};0) = \int_{\varphi^{0*}}^{\infty} r(\varphi;0)M\frac{g(\varphi)}{1-G(\varphi^{0*})}d\varphi \tag{17}$$

That is, $r(\tilde{\varphi};0)$ is the average sales that would be generated without steam power, *holding constant* the entry cut-off at its equilibrium value in the economy with steam power, φ^{0*}.

Substituting the rightmost expression in equation (16) into equation (12) yields the following welfare formula:

$$\frac{\mathbb{W}}{\mathbb{W}_a} = \left(\frac{R/M}{r(\tilde{\varphi};0)}\right)^{\frac{1}{\theta}} \tag{18}$$

To summarize, equation (18) measures the gross percentage increase in welfare from the introduction and adoption of steam power. Within the large parentheses is the ratio of average sales, R/M, to the average sales that would be realized in the economy without steam power (holding constant the entry cut-off at its equilibrium value in the economy with steam power, φ^{0*}). As this ratio is strictly positive, so too is the aggregate welfare effect from the introduction of steam power.

The coefficient in the exponent of equation (18) is the scale parameter of the Pareto distribution, θ. In the next section I discuss how firm-level data on sales and steam power adoption can be used to estimate this parameter and the term $r(\tilde{\varphi};0)$, which enable estimation of the welfare equation (18) .

4 Empirical Application

4.1 Econometric Model

In this section I develop econometric models for estimating θ and $r(\tilde{\varphi};0)$ from firm-level data. I begin by discussing the methodology for estimating $r(\tilde{\varphi};0)$. Taking the natural logarithm the revenue equation (6) after subbing in the monopolistically competitive price from equation (5) yields:

$$ln(r(\varphi;d)) = ln\left(\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{P^{\sigma-1}}{w^{\sigma}}\right) + (\sigma-1)ln(1+d\eta) + (\sigma-1)ln(\varphi) \tag{19}$$

The regression equation corresponding to equation (19) is:

$$y_i = \beta_0 + \beta_1 d_i + u_i, \tag{20}$$

where y_i is the natural log of value added, d_i is the steam power indicator variable as defined in equation (2), $\beta_0 = ln\left(((\sigma-1)/\sigma)^{\sigma-1}P^{\sigma-1}/w^{\sigma}\right)$, $\beta_1 = (\sigma-1)ln(1+\eta)$, and $u_i = (\sigma-1)ln(\varphi)$. It follows that $r(\tilde{\varphi};0)$ can be estimated by:

$$\hat{r}\left(\tilde{\varphi};0\right) = exp(\hat{\beta}_0)\frac{1}{n}\sum_{i=1}^{n}exp(\hat{u}_i) = \frac{1}{n}\sum_{i=1}^{n}exp(y_i)exp(-\hat{\beta}_1 d_i), \tag{21}$$

where $\hat{\beta}_0$ and $\hat{\beta}_0$ are the OLS estimates of β_0 and β_1, and $\hat{u}_i$ is the i^{th} residual from regression equation (20).

I now show how an estimate of θ can be obtained by estimating the parameters of the distribution of sales for the subsample of firms that had adopted steam power in 1871. Define $F\left(r\right)$ as the cumulative distribution function (CDF) of sales for the subset of firms having adopted steam power. This CDF is derived in the appendix and is given by:

$$F(r) = \begin{cases} 0 & \text{if } r < r(\varphi^{1*}; 1) \\ 1 - \left(r(\varphi^{1*}; 1)\right)^{\frac{\theta}{\sigma-1}} r^{-\frac{\theta}{\sigma-1}} & \text{if } r \geq r(\varphi^{1*}; 1) \end{cases} \tag{22}$$

Therefore, the distribution of sales for firms with steam power follows a Pareto distribution with shape parameter $\theta/(\sigma - 1)$.

For $r \geq r(\varphi^{1*}; 1)$, equation (22) can be rewritten:

$$ln(r) = ln(r(\varphi^{1*}; 1)) - \frac{\sigma - 1}{\theta} ln(1 - F(r)) \tag{23}$$

The empirical specification corresponding to equation (23) is:

$$y_i^1 = \gamma_0 + \gamma_1 ln(1 - \hat{F}_i) + \epsilon_i \tag{24}$$

where y^1 is the natural logarithm of sales for the subsample of firms with steam power, and $\hat{F}_i$ is the empirical CDF. I follow Head et al. (2014) in estimating equation (24) by the QQ regression methodology introduced by Kratz and Resnick (1996). Defining $i = 1$ as the firm with the minimum sales and $i = n$ as the firm with the maximum sales in the subsample, the empirical CDF is defined $\hat{F}_i = (i - 0.3)/(n + 0.4)$. The coefficient of interest is $\gamma_1 = -(\sigma - 1)/\theta$. Identifying θ requires an estimate of σ, the elasticity of substitution between varieties. Hsieh and Klenow (2009) note that estimates of σ in the empirical trade and industrial organization literature typically range from 3 to 10 for the manufacturing sector. Ziebarth (2013) assumes that $\sigma = 3$ in his analysis of misallocation and productivity of the US manufacturing sector in the nineteenth century. For my analysis, I follow Head et al. (2014) in assuming $\sigma = 4$ and explore the robustness of this assumption in the sensitivity analysis.

4.2 Estimation Results

For my empirical application I use firm-level data from the 1871 Canadian Census of Manufacturing sourced from the CANIND71 (2008) database, which was described in Section 2. I control for measurement error in the CANIND71 dataset by removing the bottom and top 1% of the empirical distribution of labour productivity.

The first step in the empirical application is to estimate equation (20) by OLS regression. For the empirical specification I supplement equation (20) with industry and district specific indicator variables. These indicator variables control for industry specific technological differences and cross district price variation in input markets. Under the supplemented empirical specification the rightmost equation in (21) continues to hold as the correct theoretical estimate of $\hat{r}\left(\tilde{\varphi};0\right)$. Table 3 reports the OLS regression results along with the estimate of the ratio $(R/M)/r(\tilde{\varphi};0)$. The numerator in this ratio is calculated as mean value added of firms in the sample. The estimated value of this ratio indicates that average manufacturing revenue was approximately 35.9% higher as a result of the introduction and adoption of steam power. This estimate of the scale effect of steam power is biased downwards insofar as the introduction of steam power resulted in the exit of the least productive manufacturing firms. However, as demonstrated in Section 3, my estimate of $(R/M)/r(\tilde{\varphi};0)$ is the correct statistic for calculating the aggregate welfare gain from steam power adoption as given by equation (18).

Table 3: Regression of Value Added on Steam Power Adoption

Sample	CANIND71
Power Indicator (d)	1.385
	(0.0265)
Constant	6.572
	(1.235)
$(R/M)/r(\tilde{\varphi};0)$	1.359
Observations	39,413
R-Squared	0.519

The dependent variable is the natural log of value added for Canadian manufacturing establishments in 1871. Standard errors are in parentheses. All reported regression coefficient estimates are statistically significant at the 1% level. Indicator variables for industry type and district are included in the regression. Coefficient estimates for these indicator variables are not reported but are available upon request. $R/\left(r(\tilde{\varphi};0)M\right)$ is calculated as the ratio of mean value added to the empirical estimate of $r(\tilde{\varphi};0)$, which is calculated by the rightmost equation in (21).

The QQ regression estimation results of equation (24) are reported in Table 4. I follow Head et al. (2014) in reporting results for the entire sample and several subsamples that are created by selecting the right tail of the empirical distribution at various percentiles. Head et al. (2014) note that the Pareto distribution is commonly used in the empirical trade literature because it is tractable, can be theoretically micro-founded, and empirically provides a good fit of the right tail of the observed distribution of manufacturing firm sales data. However, the left tail of this distribution is not as well approximated by the Pareto distribution, and it is not uncommon for researchers to estimate values of the scale parameter that violate the key assumption that was noted in Section 3, that $\theta > \sigma - 1$.[12] Table 4 illustrates that this assumption is indeed violated for the full sample. Theoretically valid estimates of the shape parameter, θ, are obtained when the sample is restricted to the top 50% or less (as ordered by size in value added). I follow Head et al. (2014) in estimating the welfare gains for all theoretically valid estimates of θ. The results indicate that the aggregate welfare gain from steam power adoption ranged from 5.62-9.96% depending on the estimate of θ that is used.

Table 4: QQ Regression Results for Sub-Sample using Steam

Sample	All	Top 50%	Top 25%	Top 20%	Top 15%	Top 10%	Top 5%
$\frac{\sigma-1}{\theta}$	-1.313	-0.929	-0.791	-0.745	-0.693	-0.624	-0.535
	(0.0110)	(0.00532)	(0.00610)	(0.00654)	(0.00702)	(0.00758)	(0.0101)
θ	2.285	3.229	3.793	4.027	4.329	4.808	5.607
$\%\Delta\mathbb{W}$		9.96%	8.42%	7.91%	7.34%	6.59%	5.62%
Observations	2,498	1,251	655	511	381	252	125
R-Squared	0.851	0.961	0.963	0.962	0.963	0.964	0.958

The dependent variable is the natural log of value added for the subsample of manufacturing firms that had adopted steam power in 1871. The right hand side variable is $ln(1-\hat{F}_i)$. Standard errors are in parentheses. All regression coefficient estimates are statistically significant at the 1% level. The estimates of θ and the welfare gains are calculated using $\sigma = 4$, and the estimate of $R/\left(r(\tilde{\varphi};0)M\right)$ from Table 3.

5 Conclusion

In this paper I have developed a model that explicitly accounts for the role of firm heterogeneity in the process of steam power adoption. The introduction of steam power results in three effects on aggregate welfare. First, firms with the highest productivity adopt steam power which leads to an increase in aggregate productivity and a decrease in the aggregate price index. Second, the decrease in the price index results in the exit of the firms with the lowest productivity which further increases aggregate productivity. Third, the exit of the least productive firms leads to a reduction in the set of products available to consumers, which decreases aggregate welfare as consumers gain utility from product variety. Based on this theoretical framework I have developed an empirical

[12] Head et al. (2014) derive a theoretical and empirical methodology for estimating the gains from trade under the assumption that φ follows a log normal distribution. In future work I plan to attempt to derive a similar methodology for estimating the welfare gains from the introduction and adoption of steam power under the assumption that φ follows a log normal distribution.

methodology for estimating the aggregate welfare effect of the introduction of steam power. In the empirical application of the paper I estimate that the aggregate welfare gain from the introduction of steam power in the Canadian manufacturing sector in 1871 was 5.62-9.96%.

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6 Appendix

6.1 Derivation of the Distribution of Sales for Firms with steam Power

Define $F(r)$ as the CDF of sales for the subset of firms that have adopted steam power. By definition:

$$\begin{aligned}
F(r) &= Pr\left(r(\varphi;1) \leq r \mid r(\varphi^{1*};1) \leq r(\varphi;1)\right) \\
&= \frac{Pr\left(r(\varphi^{1*};1) \leq r(\varphi;1) \leq r\right)}{Pr\left(r(\varphi;1) \geq r(\varphi^{1*};1)\right)}, \\
&= \frac{Pr\left(r(\varphi;1) \leq r\right) - Pr\left(r(\varphi;1) \leq r(\varphi^{1*};1)\right)}{Pr\left(r(\varphi;1) \geq r(\varphi^{1*};1)\right)}
\end{aligned} \tag{25}$$

Evaluating the revenue equation (6) and the monopolistically competitive price (5) yields:

$$r(\varphi;1) = K\varphi^{\sigma-1}, \quad where \;\; K = (\eta+1)^{\sigma-1} w^{2-\sigma} P^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \tag{26}$$

Using the revenue expression above and the probability expression in equation (25), the distribution $F(r)$ for $r \geq r(\varphi^{1*};1)$ is:

$$\begin{aligned}
F(r) &= \frac{Pr\left(K\varphi^{\sigma-1} \leq r\right) - Pr\left(K\varphi^{\sigma-1} \leq K\left(\varphi^{1*}\right)^{\sigma-1}\right)}{Pr\left(K\varphi^{\sigma-1} \geq K(\varphi^{1*})^{\sigma-1}\right)}, \\
&= \frac{Pr\left(\varphi \leq \left(\frac{r}{K}\right)^{\frac{1}{\sigma-1}}\right) - Pr\left(\varphi \leq \varphi^{1*}\right)}{Pr\left(\varphi \geq \varphi^{1*}\right)}, \\
&= \int_{\varphi^{1*}}^{\left(\frac{r}{K}\right)^{\frac{1}{\sigma-1}}} \frac{g(\varphi)}{1-G(\varphi^{1*})} d\varphi, \\
&= 1 - \left(r(\varphi^{1*};1)\right)^{\frac{\theta}{\sigma-1}} r^{-\frac{\theta}{\sigma-1}}
\end{aligned}$$

It follows that the CDF of sales for the subset of firms that have adopted steam power is:

$$F(r) = \begin{cases} 0 & \text{if } r < r(\varphi^{1*};1) \\ 1 - \left(r(\varphi^{1*};1)\right)^{\frac{\theta}{\sigma-1}} r^{-\frac{\theta}{\sigma-1}} & \text{if } r \geq r(\varphi^{1*};1) \end{cases} \tag{27}$$